calibration service to include WR 28-size waveguide (26.5–40.0 Gc/s). The service includes attenuation difference measurements on variable attenuators and insertion-loss measurements on fixed attenuators. This provides an attenuation calibration service over eight waveguide sizes, covering a frequency range from 2.60 to 40.0 Gc/s. Development work is in progress to extend the service in several waveguide sizes above and below the present limits to 2.6 and 40.0 Gc/s.

The Radio Standards Laboratory has discontinued the attenuation measurement of three-port and four-port waveguide directional couplers, accepting only two-port waveguide attenuators for calibration. For some years the Bureau has provided a calibration service for three-port and four-port waveguide directional couplers for use as highly stable fixed value attenuation standards. With the "in-line" type of precision fixed-waveguide attenuator becoming commercially available, the superior features of this type of attenuator make the use of the older directional coupler obsolete as a fixed attenuation standard.

II. Improved Attenuation Measurements at Microwave Frequencies

An improvement in accuracy for attenuation difference measurements on variable attenuators from 0.1 dB/10 dB to 0.05 dB/10 dB (0.5 percent of the attenuation in decibels) has been available for some time in WR 90 waveguide (8.2–12.4 Gc/s). More recently this increased accuracy has become available in four additional waveguide sizes:

WR 284 (2.6–3.95 Gc/s)
WR 187 (3.95–5.85 Gc/s)
WR 137 (5.85–8.2 Gc/s)
WR 62 (12.4–18.0 Gc/s).

An accuracy of 0.1 dB/10 dB (one percent of the attenuation in decibels) is reported for insertion-loss measurements of fixed attenuators.

This improvement in accuracy of the attenuation measurement leads to a corresponding improvement in the impedance match at the attenuator insertion points. Formerly the mismatch produced a VSWR of 1.05 or less, whereas it is possible now to hold the mismatch to a VSWR below 1.02.

III. Measurement of Waveguide Reflectors Extended to Additional Waveguide Size

Measurement of the reflection coefficient magnitude of waveguide reflectors (mismatches) in WR 62 waveguide (12.4–18.0 Gc/s) was announced by the Laboratory as a calibration service. The service was available previously in only one waveguide size, that of WR 90 (8.2–12.4 Gc/s).

Although measurement of the reflection coefficient magnitude can be made at any frequency over the range of 12.4 to 18.0 Gc/s, it should be emphasized that measurements are made at selected frequencies of 13.5, 15.0, and 17.0 Gc/s. This is done primarily for the economy and convenience of those requesting calibrations. Measurements can be made over a magnitude range of 0.025 to 1.0 with an uncertainty of plus or minus one percent. It is very essential that the flange be machined flat and smooth and be without protrusions or indentations.

IV. Calibration of RF Calorimeters Extended to 5000 Mc/s

The Radio Standards Laboratory announces that the calibration of coaxial-type RF calorimeters is now extended in frequency range to 500 Mc/s. Formerly the upper frequency limit was 400 Mc/s. Calibrations at CW power levels between 0.001 to 100 watts are made at the selected frequencies of 100, 200, 300, 400, and 500 Mc/s. Below 100 Mc/s measurements can be made at power levels extending up to 200 watts. Uncertainties in the measurements are expressed in the range of one to two percent, depending upon the stability and SWR of the calorimeter being calibrated.

V. X-Band Bolometric and Calorimetric Standards

With the availability of suitable traveling-wave tube amplifiers, it is now possible to obtain microwave energy at considerably higher power and with good stability and low noise. This source of CW microwave power is being utilized by the Radio Standards Laboratory to perform calibrations up to a power level of one watt on bolometric and calorimetric devices in the frequency range of 8.2 to 12.4 Gc/s (X-band). Formerly the power level was limited to 100 mw. Uncertainty in the measurements is expected to be no greater than one percent.

VI. Waveguide Bolometer-Coupler Units

The calibration of waveguide bolometer-coupler units[2] for use as power measurement devices at microwave frequencies requires a determination of Γ_g, the equivalent reflection coefficient looking into the output port of the coupler. Determination of the equivalent reflection coefficient by the Radio Standards Laboratory is made preferably with the bolometer unit detached from the coupler, though the determination can be made with the bolometer unit attached. It will be future practice, unless the Laboratory is specifically instructed otherwise, to separate the bolometer-coupler units to determine initially the equivalent reflection coefficient. Upon repeating a power calibration, if the measurement shows no marked change from the previous calibration, the determination of equivalent reflection coefficient will not be repeated. If there is a change in the power calibration, however, Γ_g will be redetermined with the bolometer-coupler unit intact to preserve the calibration history.

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Boulder, Colo.

[2] See, "Extension of waveguide power calibration service," *NBS Tech. News Bull.*, vol. 47, p. 141. August 1963.

Propagation in Cylindrical Waveguide Containing Inhomogeneous Dielectric

The wave equations in the cylindrical waveguide containing inhomogeneous dielectric are[1]

$$\Delta \times \Delta \times E_T - \nabla \frac{1}{\epsilon} \nabla \cdot \epsilon E_T - (\gamma^2 + \omega^2 \epsilon\mu) E_T = 0 \quad (1)$$

$$\epsilon \nabla \times \frac{1}{\epsilon} \nabla \times H_T - \nabla\nabla \cdot H_T - (\gamma^2 + \omega^2 \epsilon\mu) H_T = 0. \quad (2)$$

These equations are solved here in two cases: 1) Rectangular waveguide wherein the dielectric constant varies linearly across the broad dimensions, 2) circular waveguide wherein the dielectric constant varies quadratically with the distance from the tube axis. The surface of the waveguide is assumed to be perfectly conductive.

I. Rectangular Waveguide

A. LSE-mode[2]

Putting $E_x = 0$, $E_y = \cos\,(m\pi y/2b) \cdot X$, $\epsilon = \epsilon_0(1 - px)$ in (1):

$$\frac{d^2X}{dx^2} + (\gamma' - \epsilon' x) X = 0 \quad (3)$$

where $\gamma_i = \gamma^2 + \omega^2\epsilon_0\mu - (m\pi/2b),^2$ $\epsilon' = p\omega^2\epsilon_0\mu = (2\pi/\lambda_0)^2(n_1 - n_2)/a$. n_1, n_2 are the refraction indexes at $x = 0$, $2a$, respectively. The solutions of (3) can be written in terms of Bessel function of order $\frac{1}{3}$, $-\frac{1}{3}$. The eigenvalues γ', which are determined by the boundary condition at $x = 0$, $2a$, $E_y = 0$, are calculated from

$$J_{1/3}(\zeta^{3/2})/J_{-1/3}(\zeta^{3/2}) = J_{1/3}(\eta^{3/2})/J_{-1/3}(\eta^{3/2}) \quad \text{if} \quad \gamma' - 2\epsilon' a > 0 \quad (4)$$

$$= -I_{1/3}((-\eta)^{3/2})/I_{-1/3}(-\eta)^{3/2} \quad \text{if} \quad \gamma' - 2\epsilon' a < 0 \quad (4')$$

where

$$\zeta = (2/3\epsilon')^{2/3}\gamma', \quad \eta = (2/3\epsilon')^{2/3}(\gamma' - 2\epsilon' a).$$

The eigenvalues γ', evaluated by the use of asymptotic expansions of the Airy functions, are[3]

$$\gamma_n' \doteqdot (n\pi/2a)^2 + \epsilon' a \quad \text{if} \quad \epsilon' a \ll 1 \quad (5)$$

$$2\gamma_n'^{3/2}/3\epsilon' \doteqdot (n - 1/4)\pi \quad \text{if} \quad \epsilon' a > \gamma_n'. \quad (5')$$

B. LSM-mode

This mode can be obtained by the similar method as that of LSE-mode.

II. Circular Waveguide

In this case, the normal modes are in general combinations of TE and TM modes and are very complicated. Here calculations are made for circularly symmetrical modes (TE_{0n} and TM_{0n} modes).

C. TE_{0n} mode

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[1] R. B. Adler, "Waves on inhomogeneous cylindrical structure," *Proc. IRE*, vol. 40, pp. 339–348, March 1952.

[2] R. E. Collin, *Field Theory of Guided Waves*, New York: McGraw-Hill, 1960.

[3] D. E. Kerr, *Propagation of Short Radio Waves*, Rad. Lab. Ser. 13, New York: McGraw-Hill, 1951.

Putting $E_r=0$, $\epsilon=\epsilon_0(1-qr^2)$ in (1), the equation for E_ϕ becomes

$$\frac{d^2E_\phi}{dr^2}+\frac{1}{r}\frac{dE_\phi}{dr}+\left(\gamma''-\epsilon''r^2-\frac{1}{r}\right)E_\phi=0 \quad (6)$$

where

$$\gamma''=\gamma^2+\omega^2\epsilon_0\mu,$$
$$\epsilon''=q\omega^2\epsilon_0\mu=2(2\pi/\lambda_0)^2(n_0-n_3)/R^2.$$

n_0, n_3 are the refraction indexes at $r=0$, R, respectively. Equation (6) can be put in Whittaker's standard form of confluent hypergeometric equation by the following transformation:[4]

$$z=\sqrt{\epsilon''}r^2, \quad Y=E_\phi/r, \quad \kappa=\gamma''/4\sqrt{\epsilon''}$$

and we obtain

$$\frac{d^2Y}{dr^2}+\left(-\frac{1}{4}+\frac{\kappa}{z}\right)Y=0. \quad (7)$$

The solutions of (6) are given in terms of the Whittaker functions:

$$E_\phi=(1/\epsilon''^{1/4}r)M_{\kappa,1/2}(\sqrt{\epsilon''}r^2) \quad (8)$$

and the eigenvalues γ'', which satisfy $M_{\kappa,1/2}(\sqrt{\epsilon''}R^2)=0$, are calculated from

$$\sqrt{\epsilon''}R^2 \doteqdot (\sqrt{\epsilon''}/\gamma_n'')j_{1,n}^2(1+(\epsilon''/3\gamma_n''^2)j_{1,n}^2), \quad \text{if} \quad \sqrt{\epsilon''}R^2\ll 1 \quad (9)$$

where $j_{1,n}$ is the nth zero of $J_1(x)$,

$$(\sqrt{\epsilon''}R^2)^{-2\kappa_n}\exp(\sqrt{\epsilon''}R^2) \doteqdot -\Gamma(1-\kappa_n)/\Gamma(1+\kappa_n), \text{ if } \sqrt{\epsilon''}R^2>4n. \quad (9')$$

When $\sqrt{\epsilon''}R^2$ is large, γ'' approaches to an integer and is written as $\gamma_n''=n+\Delta$. Then (9') becomes

$$(\sqrt{\epsilon''}R^2)^{-2n}\exp(\sqrt{\epsilon''}R^2) \doteqdot \frac{(-1)^{n-1}}{n!(n-1)!\Delta}, \quad n=1,2,\cdots \quad (9'')$$

and $(1/z)^{1/2}M_{\kappa,1/2}(z)$ approaches $z^{1/2}\exp(-z/2)L_{n-1}^1(z)$.

D. TM_{0n} mode.

The wave equation for H_ϕ follows from (2):

$$\frac{d^2H}{dr^2}+\frac{1}{r}\frac{dH_\phi}{dr}-\frac{1}{\epsilon}\frac{d\epsilon}{dr}\left(\frac{dH_\phi}{dr}+H_\phi\right)+\left(\gamma''-\epsilon''r^2-\frac{1}{r^2}\right)H_\phi=0. \quad (10)$$

This equation is solved by the perturbation method and the first approximation of the solution is

$$H_\phi \doteqdot (1/\epsilon''^{1/4}r)M_{\kappa,1/2}(\sqrt{\epsilon''}r^2). \quad (11)$$

The eigenvalues γ'', which satisfy

$$(d/dz)M_{\kappa,1/2}(z)=0 \text{ at } z=\sqrt{\epsilon''}R^2,$$

are calculated from

$$\sqrt{\epsilon''}R^2 \doteqdot (\sqrt{\epsilon''}/\gamma_n'')j_{0,n}^2, \text{ if } \sqrt{\epsilon''}R^2\ll 1 \quad (12)$$

$$(\sqrt{\epsilon''}R^2)^{-2n}\exp(\sqrt{\epsilon''}R^2) \doteqdot \frac{(-1)^n}{n!(n-1)!\Delta}, \quad \text{if} \quad \sqrt{\epsilon''}R^2>4n. \quad (12')$$

From the results of the preceding calculations, we conclude that, 1) if the degree of inhomogeneity is low ($\epsilon' a\ll 1$, $\sqrt{\epsilon''}R^2\ll 1$), the configurations of the preceding modes do not differ much from those of homogeneous waveguides, 2) if the degree of inhomogeneity is high ($\epsilon' a>\gamma_n'$, $\sqrt{\epsilon''}R^2>4n$), these solutions turn into the surface waves, and the field concentrates in the region having larger dielectric constant and decays exponentially away from this region, as shown in Fig. 1.

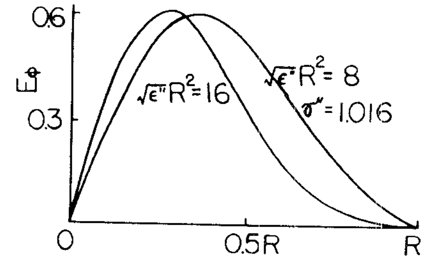

Fig. 1. Field distribution of TE_{01} mode.

These surface wave modes may propagate with very low attenuation regardless of the wall condition.

In the case when the inhomogeneity of the dielectric is caused by the temperature stratification of the air in the waveguides, the refraction index varies with the temperature as $n_1-n_2=0.93\times 10^{-6}(T_2-T_1)$, and the trapping conditions for the lowest modes, i.e., $\epsilon' a>\gamma_1'$, $\sqrt{\epsilon''}R^2>4$, can be rewritten with the temperature as $a/\lambda_0>10^2/\sqrt{T_2-T_1}$ for rectangular waveguide, $R/\lambda_0>5$ for rectangular waveguide,

$$R/\lambda_0>5\times 10^2/\sqrt{T_3-T_0}$$

$\times 10^2/\sqrt{T_3-T_0}$ for circular waveguide.

With these simplified models, we may examine the influence of the temperature stratification of the air in the oversize waveguides for transmission of submillimeter and shorter wavelength region.[5]

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